

Paper Reference(s)

6664/01

**Edexcel GCE
Core Mathematics C2
Advanced Subsidiary**

**Wednesday 9 January 2008 – Afternoon
Time: 1 hour 30 minutes**

Materials required for examination

Mathematical Formulae (Green)

Items included with question papers

Nil

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation, differentiation or integration, or have retrievable mathematical formulae stored in them.

Instructions to Candidates

Write the name of the examining body (Edexcel), your centre number, candidate number, the unit title (Core Mathematics C2), the paper reference (6664), your surname, initials and signature.

Information for Candidates

A booklet 'Mathematical Formulae and Statistical Tables' is provided.

Full marks may be obtained for answers to ALL questions.

The marks for individual questions and the parts of questions are shown in round brackets: e.g. (2).

There are 9 questions in this question paper. The total mark for this paper is 75.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.

You must show sufficient working to make your methods clear to the Examiner. Answers without working may gain no credit.

1. (a) Find the remainder when

$$x^3 - 2x^2 - 4x + 8$$

is divided by

(i) $x - 3$,

(ii) $x + 2$.

(3)

- (b) Hence, or otherwise, find all the solutions to the equation

$$x^3 - 2x^2 - 4x + 8 = 0.$$

(4)

2. The fourth term of a geometric series is 10 and the seventh term of the series is 80.

For this series, find

- (a) the common ratio,

(2)

- (b) the first term,

(2)

- (c) the sum of the first 20 terms, giving your answer to the nearest whole number.

(2)

3. (a) Find the first 4 terms of the expansion of $\left(1 + \frac{x}{2}\right)^{10}$ in ascending powers of x , giving each term in its simplest form.

(4)

- (b) Use your expansion to estimate the value of $(1.005)^{10}$, giving your answer to 5 decimal places.

(3)

4. (a) Show that the equation

$$3 \sin^2 \theta - 2 \cos^2 \theta = 1$$

can be written as

$$5 \sin^2 \theta = 3. \tag{2}$$

- (b) Hence solve, for $0^\circ \leq \theta < 360^\circ$, the equation

$$3 \sin^2 \theta - 2 \cos^2 \theta = 1,$$

giving your answer to 1 decimal place.

(7)

5. Given that a and b are positive constants, solve the simultaneous equations

$$a = 3b,$$

$$\log_3 a + \log_3 b = 2.$$

Give your answers as exact numbers.

(6)

6.

Figure 1

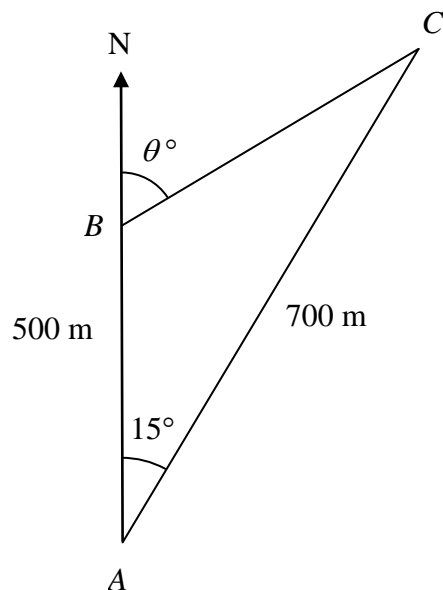


Figure 1 shows 3 yachts A , B and C which are assumed to be in the same horizontal plane. Yacht B is 500 m due north of yacht A and yacht C is 700 m from A . The bearing of C from A is 015° .

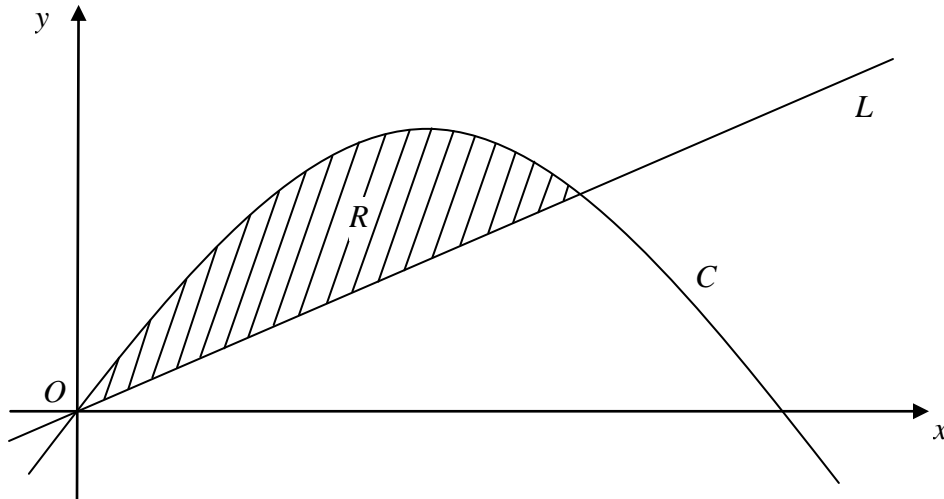
(a) Calculate the distance between yacht B and yacht C , in metres to 3 significant figures. (3)

The bearing of yacht C from yacht B is θ° , as shown in Figure 1.

(b) Calculate the value of θ . (4)

7.

Figure 2



In Figure 2 the curve C has equation $y = 6x - x^2$ and the line L has equation $y = 2x$.

(a) Show that the curve C intersects with the x -axis at $x = 0$ and $x = 6$. (1)

(b) Show that the line L intersects the curve C at the points $(0, 0)$ and $(4, 8)$. (3)

The region R , bounded by the curve C and the line L , is shown shaded in Figure 2.

(c) Use calculus to find the area of R . (6)

8. A circle C has centre $M(6, 4)$ and radius 3.

(a) Write down the equation of the circle in the form

$$(x - a)^2 + (y - b)^2 = r^2.$$

(2)

Figure 3

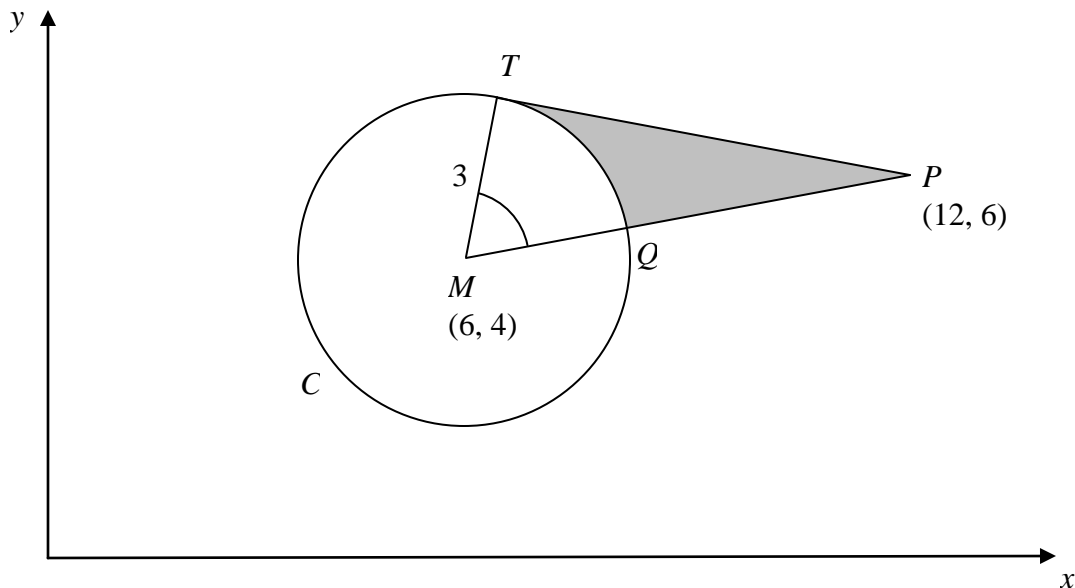


Figure 3 shows the circle C . The point T lies on the circle and the tangent at T passes through the point $P(12, 6)$. The line MP cuts the circle at Q .

(b) Show that the angle TMQ is 1.0766 radians to 4 decimal places.

(4)

The shaded region TPQ is bounded by the straight lines TP , QP and the arc TQ , as shown in Figure 3.

(c) Find the area of the shaded region TPQ . Give your answer to 3 decimal places.

(5)

9.

Figure 4

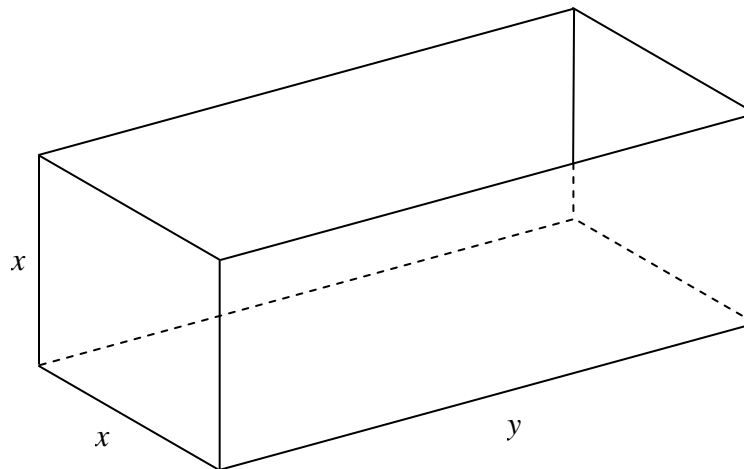


Figure 4 shows an open-topped water tank, in the shape of a cuboid, which is made of sheet metal. The base of the tank is a rectangle x metres by y metres. The height of the tank is x metres.

The capacity of the tank is 100 m^3 .

(a) Show that the area $A \text{ m}^2$ of the sheet metal used to make the tank is given by

$$A = \frac{300}{x} + 2x^2. \quad (4)$$

(b) Use calculus to find the value of x for which A is stationary. (4)

(c) Prove that this value of x gives a minimum value of A . (2)

(d) Calculate the minimum area of sheet metal needed to make the tank. (2)

TOTAL FOR PAPER: 75 MARKS

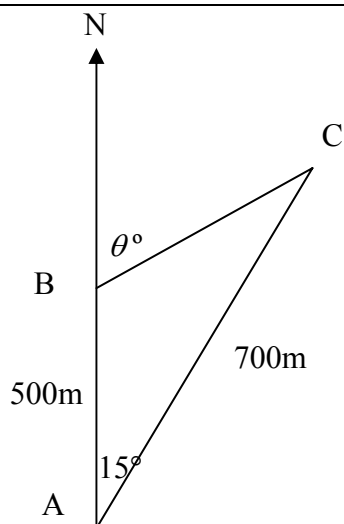
END

2.	<p>(a) Complete method, using terms of form ar^k, to find r [e.g. Dividing $ar^6 = 80$ by $ar^3 = 10$ to find r; $r^6 - r^3 = 8$ is M0] $r = 2$</p> <p>(b) Complete method for finding a [e.g. Substituting value for r into equation of form $ar^k = 10$ or 80 and finding a value for a.] $(8a = 10) \quad a = \frac{5}{4} = 1\frac{1}{4}$ (equivalent single fraction or 1.25)</p> <p>(c) Substituting their values of a and r into correct formula for sum. $S = \frac{a(r^n - 1)}{r - 1} = \frac{5}{4}(2^{20} - 1)$ (= 1310718.75) 1 310 719 (only this)</p>	<p>M1</p> <p>A1 (2)</p> <p>M1</p> <p>A1 (2)</p> <p>M1</p> <p>A1 (2) [6]</p>
Notes:	<p>(a) M1: Condone errors in powers, e.g. $ar^4 = 10$ and/or $ar^7 = 80$, A1: For $r = 2$, allow even if $ar^4 = 10$ and $ar^7 = 80$ used (just these) (M mark can be implied from numerical work, if used correctly)</p> <p>(b) M1: Allow for numerical approach: e.g. $\frac{10}{r_c^3} \leftarrow \frac{10}{r_c^2} \leftarrow \frac{10}{r_c} \leftarrow 10$</p> <p>In (a) and (b) correct answer, with no working, allow both marks.</p> <p>(c) Attempt 20 terms of series and add is M1 (correct last term 655360) If formula not quoted, errors in applying their a and/or r is M0 Allow full marks for correct answer with no working seen.</p>	
3.	<p>(a) $\left(1 + \frac{1}{2}x\right)^{10} = 1 + \underbrace{\binom{10}{1}\left(\frac{1}{2}x\right) + \binom{10}{2}\left(\frac{1}{2}x\right)^2 + \binom{10}{3}\left(\frac{1}{2}x\right)^3}_{\text{underlined}}$</p> <p>$= 1 + 5x; + \frac{45}{4}(\text{or } 11.25)x^2 + 15x^3$ (coeffs need to be these, i.e, simplified)</p> <p>[Allow A1A0, if totally correct with unsimplified, single fraction coefficients]</p> <p>(b) $(1 + \frac{1}{2} \times 0.01)^{10} = 1 + 5(0.01) + (\frac{45}{4} \text{ or } 11.25)(0.01)^2 + 15(0.01)^3$</p> <p>$= 1 + 0.05 + 0.001125 + 0.000015$</p> <p>$= 1.05114 \quad \text{cao}$</p>	<p>M1 A1</p> <p>A1; A1 (4)</p> <p>M1 A1√</p> <p>A1 (3) [7]</p>
Notes:	<p>(a) For M1 first A1: Consider underlined expression only. M1 Requires correct structure for at least two of the three terms: (i) Must be attempt at binomial coefficients. [Be generous :allow all notations e.g. ${}^{10}C_2$, even $\left(\frac{10}{2}\right)$; allow “slips”.] (ii) Must have increasing powers of x, (iii) May be listed, need not be added; <i>this applies for all marks.</i></p> <p>First A1: Requires all three correct terms but need not be simplified, allow 1^{10} etc, ${}^{10}C_2$ etc, and condone omission of brackets around powers of $\frac{1}{2}x$ Second A1: Consider as B1: 1 + 5x can score A1 on Epen, even after M0</p> <p>(b) For M1: Substituting their (0.01) into their (a) result [0.1, 0.001, 0.25, 0.025, 0.0025 acceptable but not 0.005 or 1.005] First A1 (f.t.): Substitution of (0.01) into their 4 termed expression in (a) Answer with no working scores no marks (calculator gives this answer)</p>	

<p>4. (a)</p> <p>(b)</p>	$3 \sin^2 \theta - 2 \cos^2 \theta = 1$ $3 \sin^2 \theta - 2(1 - \sin^2 \theta) = 1 \quad (\text{M1: Use of } \sin^2 \theta + \cos^2 \theta = 1)$ $3 \sin^2 \theta - 2 + 2 \sin^2 \theta = 1$ $5 \sin^2 \theta = 3 \quad \text{cso} \quad \text{AG}$ $\sin^2 \theta = \frac{3}{5}, \text{ so } \sin \theta = (\pm)\sqrt{0.6}$ <p>Attempt to solve both $\sin \theta = +..$ and $\sin \theta = -...$ (may be implied by later work)</p> $\theta = 50.7685^\circ \quad \text{awrt } \theta = 50.8^\circ \quad (\text{dependent on first M1 only})$ $\theta (= 180^\circ - 50.7685^\circ); = 129.23\dots^\circ \quad \text{awrt } 129.2^\circ$ <p>[f.t. dependent on first M and 3rd M]</p> $\sin \theta = -\sqrt{0.6}$ $\theta = 230.785^\circ \text{ and } 309.23152^\circ \quad \text{awrt } 230.8^\circ, 309.2^\circ \text{ (both)}$	<p>M1</p> <p>A1 (2)</p> <p>M1</p> <p>M1</p> <p>A1</p> <p>M1; A1 ✓</p> <p>M1A1 (7)</p> <p>[9]</p>
<p>Notes:</p>	<p>(a) N.B: AG; need to see at least one line of working after substituting $\cos^2 \theta$.</p> <p>(b) First M1: Using $5 \sin^2 \theta = 3$ to find value for $\sin \theta$ or θ</p> <p>[Allow such results as $\sin \theta = \frac{3}{5}$, $\sin \theta = \frac{\sqrt{3}}{5}$for M1]</p> <p>Second M1: Considering the – value for $\sin \theta$. (usually later)</p> <p>First A1: Given for awrt 50.8°. Not dependent on second M.</p> <p>Third M1: For $(180 - \text{candidate}' \text{ s } 50.8)^\circ$, need not see written down</p> <p>Final M1: Dependent on second M (but may be implied by answers)</p> <p>For $(180 + \text{candidate}' \text{ s } 50.8)^\circ$ or $(360 - \text{candidate}' \text{ s } 50.8)^\circ$ or equiv.</p> <p>Final A1: Requires both values. (no follow through)</p> <p>[Finds $\cos^2 \theta = k$ ($k = 2/5$) and so $\cos \theta = (\pm)...$M1, then mark equivalently]</p> <p>NB Candidates who only consider positive value for $\sin \theta$</p> <p>can score max of 4 marks: M1M0A1M1A1M0A0 – Very common.</p> <p>Candidates who score first M1 but have wrong $\sin \theta$ can score maximum</p> <p style="text-align: center;">M1M1A0M1A✓ M1A0</p> <p>SC Candidates who obtain one value from each set, e.g 50.8 and 309.2</p> <p style="text-align: center;">M1M1(bod)A1M0A0M1(bod)A0</p> <p>Extra values out of range – no penalty</p> <p>Any very tricky or “ outside scheme methods” , send to TL</p>	

5.	<p><u>Method 1</u> (Substituting $a = 3b$ into second equation at some stage)</p> <p>Using a law of logs correctly (anywhere) e.g. $\log_3 ab = 2$ M1</p> <p>Substitution of $3b$ for a (or $a/3$ for b) e.g. $\log_3 3b^2 = 2$ M1</p> <p>Using base correctly on correctly derived $\log_3 p = q$ e.g. $3b^2 = 3^2$ M1</p> <p>First correct value $b = \sqrt{3}$ (allow $3^{1/2}$) A1</p> <p>Correct method to find other value (dep. on at least first M mark) M1</p> <p>Second answer $a = 3b = 3\sqrt{3}$ or $\sqrt{27}$ A1</p> <p><u>Method 2</u> (Working with two equations in $\log_3 a$ and $\log_3 b$)</p> <p>“ Taking logs” of first equation and “ separating” $\log_3 a = \log_3 3 + \log_3 b$ M1 $(= 1 + \log_3 b)$</p> <p>Solving simultaneous equations to find $\log_3 a$ or $\log_3 b$ M1 $[\log_3 a = 1\frac{1}{2}, \log_3 b = \frac{1}{2}]$</p> <p>Using base correctly to find a or b M1</p> <p>Correct value for a or b $a = 3\sqrt{3}$ or $b = \sqrt{3}$ A1</p> <p>Correct method for second answer, dep. on first M; correct second answer M1;A1[6] [Ignore negative values]</p>	
Notes:	<p>Answers must be exact; decimal answers lose both A marks</p> <p>There are several variations on Method 1, depending on the stage at which $a = 3b$ is used, but they should all mark as in scheme.</p> <p>In this method, the first three method marks on Epen are for</p> <p>(i) First M1: correct use of log law,</p> <p>(ii) Second M1: substitution of $a = 3b$,</p> <p>(iii) Third M1: requires using base correctly on correctly derived $\log_3 p = q$</p> <p><u>Three examples of applying first 4 marks in Method 1:</u></p> <p>(i) $\log_3 3b + \log_3 b = 2$ gains second M1 $\log_3 3 + \log_3 b + \log_3 b = 2$ gains first M1 $(2\log_3 b = 1, \log_3 b = \frac{1}{2})$ no mark yet $b = 3^{1/2}$ gains third M1, and if correct A1</p> <p>(ii) $\log_3(ab) = 2$ gains first M1 $ab = 3^2$ gains third M1 $3b^2 = 3^2$ gains second M1</p> <p>(iii) $\log_3 3b^2 = 2$ has gained first 2 M marks $\Rightarrow 2\log_3 3b = 2$ or similar type of error $\Rightarrow \log_3 3b = 1 \Rightarrow 3b = 3$ does not gain third M1, as $\log_3 3b = 1$ not derived correctly</p>	

6.



(a) $BC^2 = 700^2 + 500^2 - 2 \times 500 \times 700 \cos 15^\circ$
 (= 63851.92...)
 $BC = 253$ awrt

M1 A1

A1 (3)

(b) $\frac{\sin B}{700} = \frac{\sin 15}{\text{candidate's } BC}$

M1

$\sin B = \sin 15 \times 700 / 253_c = 0.716..$ and giving an **obtuse** B (134.2°) dep on 1st M M1

$\theta = 180^\circ - \text{candidate's angle } B$ (Dep. on first M only, B can be acute) M1

$\theta = 180 - 134.2 = (0)45.8$ (allow 46 or awrt 45.7, 45.8, 45.9) A1 (4) [7]

[46 needs to be from correct working]

Notes:

(a) If use $\cos 15^\circ = \dots$, then A1 not scored until written as $BC^2 = \dots$ correctly

Splitting into 2 triangles BAX and CAX, where X is foot of perp. from B to AC

Finding value for BX and CX and using Pythagoras M1

$$BC^2 = (500 \sin 15^\circ)^2 + (700 - 500 \cos 15^\circ)^2 \quad \text{A1}$$

$$BC = 253 \quad \text{awrt} \quad \text{A1}$$

(b) Several alternative methods: (Showing the M marks, 3rd M dep. on first M)

(i) $\cos B = \frac{500^2 + \text{candidate's } BC^2 - 700^2}{2 \times 500 \times \text{candidate's } BC}$ or $700^2 = 500^2 + BC_c^2 - 2 \times 500 \times BC_c$ M1

Finding angle B M1 dep., then M1 as above

(ii) 2 triangle approach, as defined in notes for (a)

$$\tan CBX = \frac{700 - \text{value for } AX}{\text{value for } BX} \quad \text{M1}$$

Finding value for $\angle CBX$ ($\approx 59^\circ$) dep M1

$$\theta = [180^\circ - (75^\circ + \text{candidate's } \angle CBX)] \quad \text{M1}$$

(iii) Using sine rule (or cos rule) to find C first:

Correct use of sine or cos rule for C M1, Finding value for C M1

Either $B = 180^\circ - (15^\circ + \text{candidate's } C)$ or $\theta = (15^\circ + \text{candidate's } C)$ M1

(iv) $700 \cos 15^\circ = 500 + BC \cos \theta$ M2 {first two Ms earned in this case}

Solving for θ ; $\theta = 45.8$ (allow 46 or 5.7, 45.8, 45.9) M1; A1

Note: S.C. In main scheme, if θ used in place of B , third M gained immediately;

Other two marks likely to be earned, too, for correct value of θ stated.

7	<p>(a) Either solving $0 = x(6 - x)$ and showing $x = 6$ (and $x = 0$) or showing $(6,0)$ (and $x = 0$) satisfies $y = 6x - x^2$ [allow for showing $x = 6$]</p> <p>(b) Solving $2x = 6x - x^2$ ($x^2 = 4x$) to $x = \dots$ $x = 4$ (and $x = 0$) Conclusion: when $x = 4$, $y = 8$ and when $x = 0$, $y = 0$,</p> <p>(c) (Area =) $\int_{(0)}^{(4)} (6x - x^2) dx$ Limits not required Correct integration $3x^2 - \frac{x^3}{3}$ (+ c) Correct use of correct limits on their result above (see notes on limits) $[\frac{3x^2 - x^3}{3}]^4 - [\frac{3x^2 - x^3}{3}]_0$ with limits substituted $[= 48 - 21\frac{1}{3} = 26\frac{2}{3}]$ Area of triangle = $2 \times 8 = 16$ (Can be awarded even if no M scored, i.e. B1) Shaded area = \pm (area under curve - area of triangle) applied correctly $(= 26\frac{2}{3} - 16) = 10\frac{2}{3}$ (awrt 10.7)</p>	<p>B1 (1)</p> <p>M1 A1 A1 (3)</p> <p>M1 A1 M1 A1 M1 A1 (6)[10]</p>
Notes	<p>(b) In scheme first A1: need only give $x = 4$ If <i>verifying approach</i> used: Verifying $(4,8)$ satisfies both the line and the curve M1(attempt at both), Both shown successfully A1 For final A1, $(0,0)$ needs to be mentioned; accept "clear from diagram"</p> <p>(c) Alternative Using Area = $\pm \int_{(0)}^{(4)} \{(6x - x^2); -2x\} dx$ approach</p> <p>(i) If candidate integrates separately can be marked as main scheme If combine to work with = $\pm \int_{(0)}^{(4)} (4x - x^2) dx$, first M mark and third M mark $= (\pm) [2x^2 - \frac{x^3}{3}] (+ c)$ A1, Correct use of correct limits on their result second M1, Totally correct, unsimplified \pm expression (may be implied by correct ans.) A1 $10\frac{2}{3}$ A1 [Allow this if, having given $-10\frac{2}{3}$, they correct it]</p> <p>M1 <i>for correct use of correct limits</i>: Must substitute correct limits for their strategy into a changed expression and subtract, either way round, e.g. $\pm \{ []^4 - []_0 \}$ If a long method is used, e.g, finding three areas, this mark only gained for correct strategy and all limits need to be correct for this strategy. Final M1: limits for area under curve and triangle must be the same.</p> <p>S.C.(1) $\int_0^6 (6x - x^2) dx - \int_0^6 2x dx = \left[3x^2 - \frac{x^3}{3} \right]_0^6 - [x^2]_0^6 = \dots$ award M1A1 MO(limits)AO(triangle)M1(bod)A0</p> <p>(2) If, having found \pm correct answer, thinks this is not complete strategy and does more, do not award final 2 A marks</p> <p>Use of trapezium rule: M0A0MA0possibleA1for triangle M1(if correct application of trap. rule from $x = 0$ to $x = 4$) A0</p>	

8	<p>(a) $(x-6)^2 + (y-4)^2 = 3^2$</p> <p>(b) Complete method for MP: $= \sqrt{(12-6)^2 + (6-4)^2}$ $= \sqrt{40}$ or awrt 6.325</p> <p>[These first two marks can be scored if seen as part of solution for (c)]</p> <p>Complete method for $\cos \theta$, $\sin \theta$ or $\tan \theta$ e.g. $\cos \theta = \frac{MT}{MP} = \frac{3}{\text{candidate's } \sqrt{40}}$ ($= 0.4743$) ($\theta = 61.6835^\circ$) [If $TP = 6$ is used, then M0] $\theta = 1.0766$ rad AG</p> <p>(c) Complete method for area TMP; e.g. $= \frac{1}{2} \times 3 \times \sqrt{40} \sin \theta$ $= \frac{3}{2} \sqrt{31}$ ($= 8.3516..$) allow awrt 8.35</p> <p>Area (sector) $MTQ = 0.5 \times 3^2 \times 1.0766$ ($= 4.8446..$)</p> <p>Area $TPQ = \text{candidate's } (8.3516.. - 4.8446..)$ $= 3.507$ awrt [Note: 3.51 is A0]</p>	<p>B1; B1 (2)</p> <p>M1</p> <p>A1</p> <p>M1</p> <p>A1 (4)</p> <p>M1</p> <p>A1</p> <p>M1</p> <p>M1</p> <p>A1 (5)</p> <p>[11]</p>
Notes	<p>(a) Allow 9 for 3^2.</p> <p>(b) First M1 can be implied by $\sqrt{40}$ or $\sqrt{31}$</p> <p>For second M1: May find $TP = \sqrt{(\sqrt{40})^2 - 3^2} = \sqrt{31}$, then either $\sin \theta = \frac{TP}{MP} = \frac{\sqrt{31}}{\sqrt{40}}$ ($= 0.8803..$) or $\tan \theta = \frac{\sqrt{31}}{3}$ ($1.8859..$) or cos rule</p> <p>NB. Answer is given, but allow final A1 if all previous work is correct.</p> <p>(c) First M1: (alternative) $\frac{1}{2} \times 3 \times \sqrt{40 - 9}$</p> <p>Second M1: allow even if candidate's value of θ used. (Despite being given !)</p>	

9	<p>(a) (Total area) = $3xy + 2x^2$ (Vol:) $x^2y = 100$ $(y = \frac{100}{x^2}, xy = \frac{100}{x})$</p> <p>(b) Deriving expression for area in terms of x only (Substitution, or clear use of, y or xy into expression for area) (Area =) $\frac{300}{x} + 2x^2$ AG</p> <p>(c) $\frac{dA}{dx} = -\frac{300}{x^2} + 4x$ Setting $\frac{dA}{dx} = 0$ and finding a value for correct power of x, for cand. M1 [$x^3 = 75$] $x = 4.2172$ awrt 4.22 (allow exact $\sqrt[3]{75}$) $\frac{d^2A}{dx^2} = \frac{600}{x^3} + 4 = \text{positive}, > 0;$ therefore minimum</p> <p>(d) Substituting found value of x into (a) (Or finding y for found x and substituting both in $3xy + 2x^2$) [$y = \frac{100}{4.2172^2} = 5.6228$] Area = 106.707 awrt 107</p>	<p>B1 B1 M1 A1 cso (4) M1A1 A1 (4) M1;A1 (2) M1 A1 (2) [12]</p>
Notes	<p>(a) First B1: Earned for correct unsimplified expression, isw.</p> <p>(b) First M1: At least one power of x decreased by 1, and no “c” term.</p> <p>(c) For M1: Find $\frac{d^2A}{dx^2}$ and explicitly consider its sign, state > 0 or “positive” A1: Candidate’s $\frac{d^2A}{dx^2}$ must be correct for their $\frac{dA}{dx}$, sign must be + ve and conclusion “so minimum”, (allow QED, \checkmark). (may be wrong x, or even no value of x found) <u>Alternative:</u> M1: Find value of $\frac{dA}{dx}$ on either side of “$x = \sqrt[3]{75}$” and consider sign A1: Indicate sign change of negative to positive for $\frac{dA}{dx}$, and conclude minimum. OR M1: Consider values of A on either side of “$x = \sqrt[3]{75}$” and compare with “107” A1: Both values greater than “$x = 107$” and conclude minimum. Allow marks for (c) and (d) where seen; even if part labelling confused. Throughout , allow confused notation, such as dy/dx for dA/dx.</p>	